GCSE Higher Tier Mathematics

Direct and Inverse Proportion

FREE SAMPLE WORKSHEET 25
Includes Full Model Answers

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*This resource is part of the **Complete Worksheet Collection**, meticulously crafted to align with AQA, Edexcel, OCR, and WJEC specifications. It builds understanding from fundamental principles to complex, Grade 9 examination-style problems.*

Instructions

- Answer all questions.
- Your working must clearly show the **four key steps**: the proportionality statement, the equation with constant k, finding k, and the final calculation.
- The number of marks is given in brackets [ ] at the end of each question or part question.
- Calculators **are permitted** for this worksheet.
- Remember the two main forms of proportion:
  - Direct proportion: $y \propto x^n \implies y = kx^n$
  - Inverse proportion: $y \propto \frac{1}{x^n} \implies y = \frac{k}{x^n}$
- Give answers to 3 significant figures where necessary.

Key Concepts: Direct and Inverse Proportion

Proportion problems describe a relationship between two variables. The key is to translate the written description into a mathematical equation using a ”constant of proportionality,” denoted by the letter k.

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1. The Universal Four-Step Method

Method: The Universal Process for All Proportion Problems

Every proportion problem, whether direct or inverse, can be solved using this same logical process.

1. **Write the Proportionality Statement:**
   Translate the words into a statement using the proportionality symbol, $\propto$.

2. **Form the Equation:**
   Replace '$\propto$' with '$= k$' to create a general equation including the constant of proportionality, k.

3. **Find the Constant, *k*:**
   Substitute the pair of complete values you are given into the equation and solve for k.

4. **Solve the Problem:**
   Rewrite the equation with your calculated value of k, then use it to find the missing value.

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2. Direct vs. Inverse Proportion

Glossary: Translating the Language of Proportion

The wording of the problem tells you which type of relationship to use.

- **Direct Proportion Keywords:**

  "y is directly proportional to x", "y varies directly as x", "$y \propto x$"

  This means as x increases, y increases. The equation is $\boldsymbol{y = kx^n}$.

- **Inverse Proportion Keywords:**

  "y is inversely proportional to x", "y varies inversely as x", "$y \propto \frac{1}{x}$"

  This means as x increases, y *decreases*. The equation is $\boldsymbol{y = \frac{k}{x^n}}$.

Example 1 (Direct): y is directly proportional to x^2. If $y = 48$ when $x = 4$, find y when $x = 5$.

$$y \propto x^2$$
$$y = kx^2$$

$$48 = k(4)^2$$
$$48 = 16k \implies k = 3$$

$$y = 3x^2$$

$$y = 3(5)^2 = 3(25) = \mathbf{75}$$

Example 2 (Inverse): p is inversely proportional to the square root of q. If $p = 10$ when $q = 16$, find p when $q = 100$.

$$p \propto \frac{1}{\sqrt{q}}$$
$$p = \frac{k}{\sqrt{q}}$$

$$10 = \frac{k}{\sqrt{16}}$$
$$10 = \frac{k}{4} \implies k = 40$$

$$p = \frac{40}{\sqrt{q}}$$

$$p = \frac{40}{\sqrt{100}} = \frac{40}{10} = \mathbf{4}$$

Caution: ”Inversely proportional to the square of x”

Read the language very carefully. This phrase is a common source of error.

- ”inversely proportional to x” $\implies y = \frac{k}{x}$
- ”inversely proportional to the **square of** x” $\implies y = \frac{k}{x^2}$
- ”inversely proportional to the **square root of** x” $\implies y = \frac{k}{\sqrt{x}}$

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1. In the following questions, y is directly proportional to x^n.

   (a) y is directly proportional to x. If $y = 20$ when $x = 4$, find y when $x = 7$. [3]

   (b) y is directly proportional to x^2. If $y = 48$ when $x = 4$, find y when $x = 5$. [3]

   (c) p varies directly as the cube of q. If $p = 54$ when $q = 3$, find p when $q = 2$. [3]

(d) The value V is directly proportional to the square root of h. If $V = 18$ when $h = 9$, find h when $V = 30$. [3]

Total: [12]

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2. In the following questions, y is inversely proportional to x^n.

(a) y is inversely proportional to x. If $y = 5$ when $x = 6$, find y when $x = 10$. [3]

(b) y varies inversely as the square of x. If $y = 2$ when $x = 5$, find y when $x = 2$. [3]

(c) a is inversely proportional to the cube root of b. If $a = 8$ when $b = 27$, find a when $b = 64$. [3]

(d) The force F is inversely proportional to the square of the distance d. If $F = 100$ when $d = 2$, find d when $F = 4$. [3]

Total: [12]

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3. Solve the following proportion problems:

(a) The time T taken to paint a wall is inversely proportional to the number of painters N. If it takes 3 painters 8 hours, how long would it take 4 painters? [3]

(b) The resistance R of a wire is directly proportional to its length L and inversely proportional to the square of its radius r. If $R = 5$ when $L = 100$ and $r = 1$, find R when $L = 150$ and $r = 0.5$. [4]

(c) The kinetic energy E of an object is directly proportional to the square of its speed v. If $E = 100$ Joules when $v = 5$ m/s, find E when $v = 15$ m/s. [3]

(d) The intensity of light I is inversely proportional to the square of the distance d from the source. If the intensity is 8 units at a distance of 3 metres, what is the distance when the intensity is 2 units? [3]

Total: [13]

This is the end of the worksheet

1)

Reminder: Direct Proportion

”Directly proportional” means as one variable increases, the other increases. The relationship $y \propto x^n$ is always written as an equation: $\boldsymbol{y = kx^n}$, where k is the constant of proportionality. Our first step is always to use the given values to find k.

(a) Given $y \propto x$, so the equation is $y = kx$.

$$20 = k(4) \implies k = 5$$

$$y = 5x = 5(7) = \mathbf{35}$$

(b) Given $y \propto x^2$, so the equation is $y = kx^2$.

$$48 = k(4^2) = 16k \implies k = 3$$

$$y = 3x^2 = 3(5^2) = 3(25) = \mathbf{75}$$

(c) Given $p \propto q^3$, so the equation is $p = kq^3$.

$$54 = k(3^3) = 27k \implies k = 2$$

$$p = 2q^3 = 2(2^3) = 2(8) = \mathbf{16}$$

(d) Given $V \propto \sqrt{h}$, so the equation is $V = k\sqrt{h}$.

$$18 = k\sqrt{9} = 3k \implies k = 6$$

$$\begin{aligned} 30 &= 6\sqrt{h} \\ 5 &= \sqrt{h} \\ h &= 5^2 = \mathbf{25} \end{aligned}$$

2)

Reminder: Inverse Proportion

"Inversely proportional" means as one variable increases, the other decreases. The relationship $y \propto \frac{1}{x^n}$ is written as an equation: $\boldsymbol{y = \frac{k}{x^n}}$. The constant k is still found in the same way.

(a) Given $y \propto \frac{1}{x}$, so the equation is $y = \frac{k}{x}$.

$$5 = \frac{k}{6} \implies k = 30$$

$$y = \frac{30}{10} = \mathbf{3}$$

(b) Given $y \propto \frac{1}{x^2}$, so the equation is $y = \frac{k}{x^2}$.

$$2 = \frac{k}{5^2} \implies k = 50$$

$$y = \frac{50}{2^2} = \frac{50}{4} = \mathbf{12.5}$$

(c) Given $a \propto \frac{1}{\sqrt[3]{b}}$, so the equation is $a = \frac{k}{\sqrt[3]{b}}$.

$$8 = \frac{k}{\sqrt[3]{27}} = \frac{k}{3} \implies k = 24$$

$$a = \frac{24}{\sqrt[3]{64}} = \frac{24}{4} = \mathbf{6}$$

(d) Given $F \propto \frac{1}{d^2}$, so the equation is $F = \frac{k}{d^2}$.

$$100 = \frac{k}{2^2} \implies k = 400$$

$$4 = \frac{400}{d^2}$$
$$d^2 = \frac{400}{4} = 100$$
$$d = \mathbf{10}$$

3)

Deeper Insight: Combined Variation

Sometimes a variable is proportional to multiple other variables. For example, if $R \propto L$ and $R \propto \frac{1}{r^2}$, we combine them into a single equation: $\boldsymbol{R = \frac{kL}{r^2}}$.

(a) Given $T \propto \frac{1}{N}$, so $T = \frac{k}{N}$.

$$8 = \frac{k}{3} \implies k = 24$$

$$T = \frac{24}{4} = \mathbf{6} \text{ hours}$$

(b) Given $R \propto \frac{L}{r^2}$, so $R = \frac{kL}{r^2}$.

$$5 = \frac{k(100)}{1^2} \implies k = 0.05$$

$$R = \frac{0.05(150)}{(0.5)^2} = \frac{7.5}{0.25} = \mathbf{30}$$

(c) Given $E \propto v^2$, so $E = kv^2$.

$$100 = k(5^2) \implies k = 4$$

$$E = 4(15^2) = 4(225) = \mathbf{900} \text{ Joules}$$

(d) Given $I \propto \frac{1}{d^2}$, so $I = \frac{k}{d^2}$.

$$8 = \frac{k}{3^2} \implies k = 72$$

$$2 = \frac{72}{d^2}$$
$$d^2 = 36$$
$$d = \mathbf{6} \text{ metres}$$